Stokes' Theorem

1. Let $\vec{F}(x,y,z) = \langle -y, x, xyz\rangle$ and $\vec{G} = \operatorname{curl}\vec{F}$. Let $\mathcal{S}$ be the part of the sphere $x^2+y^2+z^2 = 25$ that lies below the plane $z = 4$, oriented so that the unit normal vector at $(0,0,-5)$ is $\langle 0,0,-1\rangle$. Use Stokes' Theorem to find $\displaystyle\iint_{\mathcal{S}} \vec{G}\cdot d\vec{S}$.

2. Let $\vec{F}(x,y,z) = \langle -y, x, z\rangle$. Let $\mathcal{S}$ be the part of the paraboloid $z = 7 - x^2 - 4y^2$ that lies above the plane $z = 3$, oriented with upward pointing normals. Use Stokes' Theorem to find $\displaystyle\iint_{\mathcal{S}} \operatorname{curl}\vec{F}\cdot d\vec{S}$.

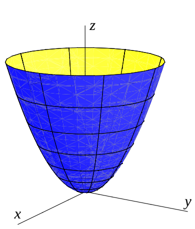
3. The plane $z = x + 4$ and the cylinder $x^2 + y^2 = 4$ intersect in a curve C. Suppose C is oriented counterclockwise when viewed from above. Let $\vec{F}(x,y,z) = \langle x^3 + 2y, \sin y + z, x + \sin z^2\rangle$. Evaluate the line integral $\displaystyle\int_C \vec{F}\cdot d\vec{r}$.

4. Let C be the oriented curve parameterized by $\vec{r}(t) = \langle \cos t, \sin t, 8 - \cos^2 t - \sin t \rangle$, $0 \le t < 2\pi$, and let $\vec{F}$ be the vector field $\vec{F}(x, y, z) = \left\langle z^2 - y^2, -2xy^2, e^{\sqrt{z}} \cos z \right\rangle$. Evaluate $\displaystyle\int_C \vec{F} \cdot d\vec{r}$.

5. Let C be the curve of intersection of $2x^2 + 2y^2 + z^2 = 9$ with $z = \frac{1}{2}\sqrt{x^2 + y^2}$, oriented counterclockwise when viewed from above, and let $\vec{F}(x, y, z) = \langle 3y, 2yz, xz^3 + \sin z^2 \rangle$. Evaluate $\displaystyle\int_C \vec{F} \cdot d\vec{r}$.

6. The two surfaces shown have the same boundary. Suppose they are both oriented so that the light side is the "positive" side. Is the following reasoning correct? "Since $\mathcal{S}_1$ and $\mathcal{S}_2$ have the same (oriented) boundary, the flux integrals $\displaystyle\iint_{\mathcal{S}_1} \vec{G} \cdot d\vec{S}$ and $\displaystyle\iint_{\mathcal{S}_2} \vec{G} \cdot d\vec{S}$ must be equal for any vector field $\vec{G}$. Therefore, you can compute any flux integral using the simpler surface."

$\mathcal{S}_1$



$\mathcal{S}_2$